

MAS231

Assignment 4

Geometry II
Due on 11 February 2008

*Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the Green Post-Box on the ground floor in the Maths building before **16:00** on Monday.*

1. Consider $\gamma(t) = (t, \cosh(t), 0)$, the Catenary curve.
 - (a) Define the curvature K and torsion T of regular curve γ and state what it means for K, T to be parametrization invariant.
 - (b) Find the arc length of γ starting from initial time $t_0 = 0$ by using the fact that $\sinh^2 \theta + 1 = \cosh^2 \theta$.
 - (c) Show that a unit speed reparametrization of $\gamma(t)$ is $\tilde{\gamma}(s) = (\sinh^{-1}(s), \sqrt{1+s^2}, 0)$.
2.
 - (a) Compute the curvature $K(t)$ and torsion $T(t)$ of the twisted curve $\gamma(t) = (t, t^2, t^3)$ for $t \in (0, \infty)$. How does the torsion behave for large t ?
 - (b) State the formula for the reparametrized twisted curve $\tilde{\gamma}(\tilde{t})$ when $\phi(\tilde{t}) = e^{\tilde{t}}$ and determine $\tilde{\alpha}$ and $\tilde{\beta}$.
 - (c) Compute the curvature $K(\tilde{t})$ for the curve $\tilde{\gamma}(\tilde{t})$ in part (b) and verify that K is preserved under this reparametrization.
 - (d) Compute the orthogonal vectors τ, ν and β for the twisted curve $\gamma(t) = (t, t^2, t^3)$.
3. Suppose you are given two orthogonal unit vectors τ, β (this is all you need to know about them for this question). In your answers to the following questions you may quote any identities for scalar and vector products that you need.
 - (a) Show that ν defined by $\nu = \beta \times \tau$ is again a unit vector.
 - (b) Show that $\tau = \nu \times \beta$ and that $\beta = \tau \times \nu$.
4. Consider the unit speed curve

$$\gamma(t) = \left(\frac{4}{5} \cos t, 1 - \sin t, -\frac{3}{5} \cos t \right).$$

- (a) Compute the curvature K and torsion T for γ .
- (b) Compute the orthogonal vectors τ, ν and β for γ using their formulae for unit speed curves.
- (c) Is $\gamma(t)$ contained in a plane? If it is, then find a vector orthogonal to that plane.